A General Approach to Modeling Jet-Turbulence Interaction Problems using Rapid-distortion Theory

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ABSTRACT

In this talk we present recent developments in the Rapid-distortion theory of turbulence when applied to the prediction of sound radiated by non-homogeneous turbulence interacting with a trailing edge of a semi-infinite flat plate positioned parallel to the level curves of an otherwise arbitrary mean flow field. The latter problem has received much attention in Aero-acoustics research community and is a canonical representation of jet installation effects.

1. Introduction

Rapid-distortion theory (RDT) uses linearized equations to analyze rapid changes in turbulent flows such as those that occur when the flow interacts with solid surfaces. It applies whenever the turbulence intensity is small and the length (or time) scale over which the changes take place is short compared to the length (or time) scale over which the turbulent eddies evolve. When interpreted asymptotically, these assumptions imply, among other things, that it is possible to identify a distance that is very (infinitely) large on the scale of the interaction, but still small on the scale over which the turbulent eddies evolve. The assumptions also imply that the resulting flow is inviscid and non-heat conducting and is, therefore, governed by the Linearized Euler Equations, i.e., the Euler equations linearized about an arbitrary, usually steady, solution (the base flow) to the nonlinear equations.

Fig. 1 Proposed aircraft where jet/edge interaction play an important role.

An important consequence of the disparate length scales is that upstream boundary conditions can be imposed infinitely far upstream in a region where the flow is undisturbed by the interaction. The two arbitrary convected quantities do not decay at upstream infinity and can, therefore, be determined from these conditions. But a major problem with this is that these quantities do not correspond to physically measurable variables and the causal RDT solutions for these variables decay at large upstream distances. Goldstein *et al*. [1] showed that since appropriate gradients of these quantities do not decay at upstream infinity, these latter arbitrary convected quantities can be related to measurable flow variables thereby developing a set of physically realizable upstream boundary conditions for planar mean flows and also flows of arbitrary cross-section (Goldstein, Leib & Afsar, 2019 [2]).

Our aim is here is to review the theoretical development of the problem based on its application to the installation noise problem of the type shown in Fig. 1. In the talk, we also consider how the basic RDT problem can be used to study the evolution of turbulence undergoing interaction.

2. Fundamental Solution to the RDT Equations

Goldstein *et al*. [1] show that the pressure fluctuation produced at the observation point, $\boldsymbol{x}=\{x_1,x_2,x_3\}$, by the interaction of the arbitrary convected disturbance $\tilde{\omega}_c\left(\tau-y_1/U(\boldsymbol{y}_T),\boldsymbol{y}_T\right)$ with solid surfaces embedded in the transversely sheared mean flow of an inviscid, non-heat conducting ideal gas is given by

$$p'(\boldsymbol{x},t)=\int_{-T}^{T}\int_{V}G(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)\,\tilde{\omega}_c\left(\tau-\frac{y_1}{U(\boldsymbol{y}_T)},\boldsymbol{y}_T\right)d\boldsymbol{y}d\tau \qquad (1)$$

where $\boldsymbol{y}=\{y_1,y_2,y_3\}$ is a Cartesian coordinate system with streamwise and transverse components, y_1 and $\boldsymbol{y}_T=\{y_2,y_3\}$ respectively, $\tilde{\omega}_c\left(\tau-y_1/U(\boldsymbol{y}_T),\boldsymbol{y}_T\right)$ can be specified as an upstream boundary condition and $G(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)$ denotes the Green's function that satisfies the inhomogeneous Rayleigh equation:

$$L\,\mathrm{G}(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)=\frac{D_0^3}{Dt^3}\delta(\boldsymbol{y}-\boldsymbol{x})\delta(\tau-t) \qquad (2)$$

where $D/Dt=\partial/\partial t+U\partial/\partial x_1$ and:

$$L\equiv\frac{D_0}{D\tau}\left(\frac{\partial}{\partial y_i}c^2\frac{\partial}{\partial y_i}-\frac{D_0^2}{D\tau^2}\right)-2\frac{\partial U}{\partial y_j}\frac{\partial}{\partial y_1}c^2\frac{\partial}{\partial y_j} \qquad (3)$$

is the Rayleigh operator. The convective derivative in (3) is $D/D\tau=\partial/\partial\tau+U\partial/\partial y_1$, the mean flow gradient in (3) is $\partial U(\boldsymbol{y}_T)/\partial y_i$ and $c^2=c^2(\boldsymbol{y}_T)$ denotes the mean sound speed. The solid surfaces $S(\boldsymbol{y})$ bound volume $V(\boldsymbol{y})$ in formulae (1) & (2) can be finite, semi-infinite or infinite in the streamwise direction but its generators must be parallel to the level curves of the mean velocity field. The Green's function, $G(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)$, now satisfies the homogeneous boundary condition $\Gamma(\boldsymbol{y},\tau\,|\,\boldsymbol{x},t)=0$ for $\boldsymbol{y}\in S$ where the scalar field, Γ, is

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determined to within an arbitrary convected quantity by the boundary condition:

$$\frac{D_0^2\Gamma(\boldsymbol{y},\tau\mid\boldsymbol{x},t)}{D\tau^2}\equiv\hat{n}_j c^2\frac{\partial G(\boldsymbol{y},\tau\mid\boldsymbol{x},t)}{\partial y_j}, \tag{4}$$

which reduces to the usual zero normal derivative boundary condition on the (impermeable) plate surface S present in the flow for the canonical RDT problem in Fig. 2. Downstream of the trailing edge in Fig. 2, the Green's function $G(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ must satisfy the jump conditions $\Delta[G]=\Delta[\Gamma]=0$ for $\boldsymbol{y}_T\in S_0$ across the resulting downstream wakes (or vortex sheets) where S_0 denotes the surfaces of discontinuity and $\Delta[\bullet]$ denotes the jump in $[\bullet]$ across these surfaces. The mean velocity profiles can be discontinuous across the wakes which can then support additional spatially growing instability waves that can be generated by imposing a Kutta condition at the trailing edge or suppressed by imposing a boundedness requirement.

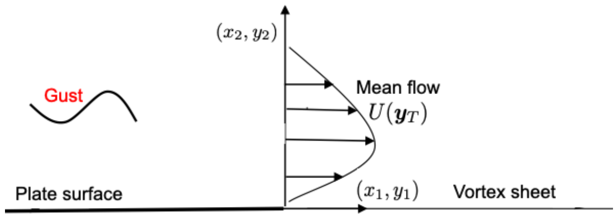

Fig. 2 Canonical Rapid-distortion theory problem.

The density-weighted transverse velocity perturbation $u_\perp$ is defined via: $|\boldsymbol{\nabla}U|u_\perp\equiv(\partial U/\partial y_i)\tilde{u}_i$ where the transverse mass flux perturbation is $\boldsymbol{u}_T=\rho(v_2',v_3')$ with $\rho=\rho(\boldsymbol{y}_T)$ being the mean flow density and $\boldsymbol{v}_T'=(v_2',v_3')$ being the actual transverse velocity perturbation. The pseudo-density-weighted transverse velocity perturbation $\tilde{u}_i$ is given by integral solution:

$$\tilde{u}_i=-\int_{-T}^{T}\int_V G_i(\boldsymbol{y},\tau\mid\boldsymbol{x},t)\tilde{\omega}_c\left(\tau-\frac{y_1}{U(\boldsymbol{y}_T)},\boldsymbol{y}_T\right)dyd\tau \text{ for } i=2,3 \tag{5}$$

with $G_i(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ determined in terms of the three-dimensional gradient of $G(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ by

$$\frac{D_0}{Dt}G_i(\boldsymbol{y},\tau\mid\boldsymbol{x},t)=-\frac{\partial}{\partial x_i}G(\boldsymbol{y},\tau\mid\boldsymbol{x},t), \quad \text{for } i=1,2,3 \tag{6}$$

3. RDT Solution Strategy

The simplest way to determine the acoustic radiation using (1) is to split the Green's function up into a hydrodynamic component (that does not generate any acoustic waves at subsonic Mach numbers) and a non-hydrodynamic component that corresponds to the acoustic waves that propagate to the far-field when inserted into (1). Mathematically, this can be accomplished by dividing the Rayleigh equation Green's function that appears in the solution (1) into two components: $G(\boldsymbol{y},\tau\mid\boldsymbol{x},t)=G^{(0)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)+G^{(s)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ where $G^{(0)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ denotes a particular solution of (2) which can either be defined on all space or, be required to satisfy appropriate boundary conditions on a streamwise extension (i.e. the vortex sheet) of the bounding surface S that extends from minus to plus infinity in the streamwise direction. This decomposition implies $G_i(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ in (5) also decomposes in a similar way and therefore the pressure fluctuation in (1), decomposes as $p'(\boldsymbol{x},t)=p'^{(0)}(\boldsymbol{x},t)+p'^{(s)}(\boldsymbol{x},t)$ where $p'^{(0)}(\boldsymbol{x},t)$ which is given by (1) and (2) with $G(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ replaced by $G^{(0)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$, does not produce any acoustic radiation at subsonic Mach numbers and can, therefore, be identified with the hydrodynamic component of the unsteady motion. On the other hand, the 'scattered component', $G^{(s)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$, satisfies the homogeneous Rayleigh's equation along with appropriate inhomogeneous boundary and jump conditions on the streamwise discontinuous surfaces and S and S_0. The corresponding 'scattered solution' $p'^{(s)}(\boldsymbol{x},t)$ therefore, accounts for all of the acoustic components of the motion.

4. Concluding Remarks

The basic solution procedure of a problem that calls for the use of RDT involves the following: (a). Take temporal/streamwise Fourier transforms of (1); (b). Insert the additive decomposition of the Green's function above; (c) determine 'gust solution' Green's function, $G^{(0)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ that is subject to boundary conditions far upsteam of the streamwise discontinuity (Fig. 2) for the canonical scattering problem we are concerned with here; (d). Solve a Wiener-Hopf problem to determine $G^{(s)}(\boldsymbol{y},\tau\mid\boldsymbol{x},t)$ in which $G^{(0)}$ enters as a gust-induced boundary coundition when inserted into (5) and using $\Delta[G]=\Delta[\Gamma]=0$ for $\boldsymbol{y}_T\in S_0$. In the talk acompanying this paper, I shall discuss how the formalism can be extended to consider jets of arbitrary cross-section that possess otherwise arbitrary mean flow fields $U=U(y_2,y_3)$ and in particular, for these flows, how one relates $\tilde{\omega}_c$ to the upstream physics so that the gust solution, found when $G^{(0)}$ is inserted in (5), is function of measurable quantities [3].

References

[1] M. E. Goldstein, S. J. Leib, M. Z. Afsar, *J. Fluid Mech.*, 824, (2017), 477—512.
[2] M. E. Goldstein, S. J. Leib, M. Z. Afsar, Submitted.
[3] M. Z. Afsar, S. J. Leib, R. E. Bozak, *J. Sound & Vib.*, 386, (2017), 177-207.